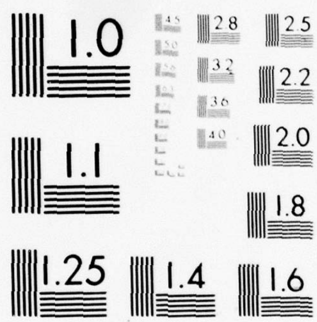
AD-A054 554 WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER F/G 12/1
PLANNING IN INTERCONNECTED POWER SYSTEMS. AN EXAMPLE OF TWO-STA--ETC(U)
MAR 78 A PREKOPA DAAG29-75-C-0024
UNCLASSIFIED MRC-TSR-1835 NL

1 OF 1
AD
A054554

END
DATE
FILMED
7-78
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

12

5c

MRC Technical Summary Report #1835

PLANNING IN INTERCONNECTED POWER SYSTEMS: AN EXAMPLE OF TWO-STAGE PROGRAMMING UNDER UNCERTAINTY

András Prékopa

Mathematics Research Center
University of Wisconsin–Madison
610 Walnut Street
Madison, Wisconsin 53706

March 1978

(Received January 31, 1978)

Sponsored by

U.S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

PLANNING IN INTERCONNECTED POWER SYSTEMS: AN EXAMPLE OF TWO-STAGE PROGRAMMING UNDER UNCERTAINTY

András Prékopa

Technical Summary Report #1835
March 1978

ABSTRACT

The problem of planning in interconnected power systems is formulated as a stochastic programming model, a variant of the model: two stage programming under uncertainty. In our case the solvability of the "second stage problem" is required only for a probability near unity.

AMS(MOS) Subject Classifications: 90C15, 90C25, 62C99, 69.52.

Key Words: Stochastic optimization, Logconcave measures, Power systems.

Work Unit Number 5 - Mathematical Programming and Operations Research.

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

Sec 1473

SIGNIFICANCE AND EXPLANATION

Mathematical optimization techniques are now widely used in business, government service, and the armed forces. A typical problem involves finding the cheapest method of providing certain services, using various available sources of supply, under given constraints, e.g. on the level of production of individual sources. Problems where we know exactly the sources of supply and their limitations, are called deterministic. Such problems have led to intensive development of mathematical optimization techniques such as linear and nonlinear programming.

In most situations there is some uncertainty as regards the sources of supply and their limitations. Such problems are called stochastic. They have received much less attention than deterministic problems, partly because they are much more difficult.

The type of problem considered in this paper will be described in terms of electrical power systems; it should be clear that the model covers very many other situations. Suppose that the power systems are interconnected so as to form a network. If a system cannot serve the demand of its own area because of (random) excess demand and/or (random) deficiency in the electricity generation, then the other systems help to meet the total demand. How can we design the cheapest possible system so as to reach a prescribed (high) reliability level?

The problems considered in this paper are based on the following underlying deterministic model:

$$\text{Minimize} \quad (c'u + d'v)$$
$$\text{subject to} \quad Au \geq b, \; Tu + Mv \geq \xi \; .$$

When the ξ is assumed to be random, the model is formulated as a two-stage problem in which we first decide on u, then choose the random variable ξ, then find v. The variable u represents supply, ξ represents random demand plus deficiency in various areas, and v represents dispatch power.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

PLANNING IN INTERCONNECTED POWER SYSTEMS:

AN EXAMPLE OF TWO-STAGE PROGRAMMING UNDER UNCERTAINTY

András Prékopa

1. Two-Stage Programming under Uncertainty

Vectors and matrices will be denoted by small resp. capital latin letters. As in many papers and textbooks on probability, random vectors will be denoted by greek letters. The prime means transpose.

The stochastic programming model: two stage programming under uncertainty has been introduced by Dantzig and Madansky [3]. Before formulating this, we remark that stochastic programming models are usually formulated in such a way that first we start from a deterministic problem that we call the underlying deterministic problem, then observe that some of the parameters appearing in the model are random and finally formulate a decision principle taking into account the underlying problem and the probability distribution of the random variables involved.

In case of the model formulated by Dantzig and Madansky, the underlying deterministic problem is the following

$$\text{minimize} \quad (c'u + d'v)$$

$$\text{subject to}$$

$$Au \geqq b \,, \tag{1.1}$$

$$Tu + Mv \geqq \xi \,.$$

Having observed that ξ is random, the model of two-stage programming under uncertainty is formulated as the following non-linear programming problem

$$\text{minimize} \quad \{c'u + E[\mu(u,\xi)]\}$$

$$\text{subject to}$$

$$Au \geqq b \,, \tag{1.2}$$

$$u \in K \,,$$

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

where K is the set of those u vectors which have the property that for every ξ in the set of its possible values there exists at least one v (depending on u and ξ) such that $Tu + Mv \geqq \xi$ and $\mu(u,\xi)$ is the optimum value of the following linear programming problem (where u and ξ are fixed for a moment, $u \in K$, $Au \geqq b$):

$$\text{minimize } d'v$$

$$\text{subject to}$$

$$Tu + Mv \geqq \xi \quad . \tag{1.3}$$

This is called the second stage problem and (1.2) the first stage problem. In the practical application of this model first we decide on u on the basis of Problem (1.2), then ξ is realized and finally we decide on v on the basis of Problem (1.3). We solve the first stage problem only once but then the "experiment" producing the random variable ξ is "performed" many times independently of each other. Thus we solve the second stage problem many times and $E(\mu(u,\xi))$ is interpreted as the arithmetic mean of the obtained optima. If ξ has a discrete distribution, then we can obtain at once the first stage and the second stage optimal solutions [3].

We recall a few facts concerning this model. We also give the main ideas of the corresponding proofs. For more detailed explanation and references the reader may consult the book of Kall [6].

K is a convex polyhedron [14]. In fact letting ξ vary in the entire space from where ξ takes its (vector) values, the set of all those u,ξ vectors for which a v can be found satisfying $Au + Bv \geqq \xi$, forms a convex polyhedron. Assume that it is given by the linear inequalities

$$G\xi \leqq Hu \quad .$$

Now, if h is the vector consisting of the components

$$\sup G\xi$$

where the supremum is taken with respect to all realizations of ξ (a zero-probability set can be neglected, of course), then clearly

$$K = \{u \mid Hu \geqq h\} \quad ;$$

this proves the assertion.

For every fixed ξ the second stage optimum value $\mu(u,\xi)$ is a convex function of the variable u. This follows from the fact that the second constraint in Problem (1.3) can be written as $Mv \geqq \xi - Tu$ and that the optimum value of a linear programming problem is a convex function of the right hand side vector provided we have an objective function to be minimized. If the set of possible values of ξ is convex, then $\mu(u,\xi)$ is convex both in u and ξ.

If $E(\xi)$ exists and $\mu(u,\xi)$ is finite for every feasible u and every ξ in its set of possible values, then $E(\mu(u,\xi))$ exists and is a convex function of u. In fact the dual of Problem (1.3) contains the components of ξ as coefficients of variables in the objective function. If such an objective function is finite for every ξ and $E(\xi)$ exists then - as it is easy to see - the expectation of the objective function also exists. The convexity of $E(\mu(u,\xi))$ follows from the convexity of $\mu(u,\xi)$ for every fixed ξ.

In [9] we gave a variant for the model: two stage programming under uncertainty by dropping the condition that the second stage problem must have feasible solution for every ξ. We require the solvability of the second stage problem only by a certain probability. The formulation of this model was motivated by the fact that in many applications (e.g. in problems of engineering design) the working of the system cannot be ensured by probability 1 but only by a somewhat lower probability. In other words, in many cases Problem (1.3) will not have feasible solution for every possible ξ. If e.g. ξ has a nondegenerated multivariate normal distribution, then the set of possible values of ξ is the entire space and the requirement of the solvability of the second stage problem for every ξ limits the structure of the matrix M a great deal.

The set of all u,ξ vectors for which there exist v satisfying the constraints of Problem (1.1), can be given by a system of linear inequalities with variables u and ξ. Let us write it (as before) in the following form

$$G\xi \leqq Hu \quad . \tag{1.4}$$

Then our new first stage problem is the following

$$\text{minimize} \quad \{c'u + E[\mu(u,\xi)]\}$$

$$\text{subject to}$$

$$Au \geqq b \ ,$$

$$P(G\xi \leqq Hu) \geqq p \ ,$$

where p is a prescribed (high) probability and (u,ξ) is the random optimum value of the new second stage problem:

$$\text{(1.5)} \qquad \begin{array}{l} \text{minimize} \quad (d'v + t'z) \\ \text{subject to} \\ Tu + Mv + z \geqq \xi \ . \end{array}$$

Here u is fixed and such that it satisfies the constraint $Au \geqq b$. The term $d'v$ in the objective function expresses the cost of the second stage activity and $t'z$ is the cost of infeasibility of the original second stage problem (1.3). We assume that t is chosen in such a way that all of its components are positive thus making $z = 0$ whenever to the given u and ξ there exists v such that $Tu + Mv \geqq \xi$.

In our problem of power systems planning we shall use separable nonlinear function $\sum_j t_j(z_j)$ instead of the linear function $t'z$. It is not difficult to show that if the functions $t_j(z_j)$ are non-decreasing and convex, then $\mu(u,\xi)$ is still convex in u for every fixed ξ.

The model of two-stage programming under uncertainty with probabilistic constraint on the solvability of the second stage problem will be shown in this paper an important, practical model construction.

2. The Problem of Planning in Interconnected Power Systems

Finding the reliability of interconnected power systems, or in other terms, finding the Loss of Load Probability (LOLP) is an important subject in power systems engineering and has an extensive literature [1]. Planning under reliability constraint seems to be a recent idea. One approach has been presented by Scherer and Joe [12]. They assume that each plant has only one generating unit that can be up or down. Using mixed integer programming they find the sizes of the plants subject to a probabilistic constraint. The interconnection is disregarded. A discussion of the previous planning models is also presented. Our approach will be different, the interconnection of the power systems plays an important role here. Below we list our assumptions and notations.

Assumptions

1. The power systems are connected in such a way that (mathematically) they form a network (see the definition in Section 3).

2. The system must have a realiability level $1 - \text{LOLP} \geqq p$ where p is a probability (near unity in practice) prescribed by ourselves.

3. We may increase the power generating capacities of the existing systems and the tie line capacities among them for cost.

4. No distinction will be made between capacity and reserve capacity or demand and deficiency. We shall simply use the terms "capacity" and "demand".

5. Each power system can help others to the extent of its surplus power generation capacity and the tie line capacities. The tie line between systems a and b is not necessarily the same as that between b and a, further these two may have different capacities.

6. Power dispatching and outage cost money. The first one will be assumed to be linear while the seond one nonlinear of a fixed charge type.

Notations

N number of power systems in the pool;

x_j generating capacity of the jth system, to be determined;

y_{hk} capacity of the tie line between system h and system k, to be determined;

i_j bus power at the jth system; it can take on positive, zero and negative values depending whether plant j helps other(s), serves only its own area or receives help from others;

i_{hk} power dispatched from system h to system k; it is assumed that $i_{hk} = -i_{kh}$;

$$i^+_{hk} = \begin{cases} i_{hk} & \text{if } i_{hk} \geqq 0 , \\ 0 & \text{if } i_{hk} < 0 , \end{cases}$$

$$i^-_{hk} = \begin{cases} 0 & \text{if } i_{hk} \geqq 0 , \\ -i_{hk} & \text{if } i_{hk} < 0 ; \end{cases}$$

p prescribed lower level for 1-LOLP ;

ξ_j random demand for electric power (including deficiency) at the jth system;

$c_j(x)$ cost function of the generating capacity at the jth system;

$c_{hk}(y)$ cost function of the interconnection capacity between system h and system k;

d_{hk} cost of dispatching one unit of power from system h to system k (dispatching cost is assumed to be a linear function);

$t_j(z)$ cost of outage of magnitude z at system j when no other system is able to assist to meet total demand;

$x_j^{(\ell)}, x_j^{(u)}$ prescribed lower resp. upper bound for x_j ;

$y_{hk}^{(\ell)}, y_{hk}^{(u)}$ prescribed lower resp. upper bound for y_{hk}.

Let $A(x,y,\xi)$ denote the event that for fixed x and y the total demand in the pool can be met by a suitable power dispatch. The event $A(x,y,\xi)$ will be given in terms of linear inequalities in the next section.

We define further a dispatch function $d(x,y,\xi)$ that is the cost of the minimum cost dispatch necessary to meet the total demand in the pool provided it is possible i.e. $A(x,y,\xi)$ occurs. In general, no formula can be given for $d(x,y,\xi)$; for every x,y,ξ the minimum cost dispatch (flow) can be determined algorithmically.

If $A(x,y,\xi)$ does not occur, then instead of the area demands $\xi_j - x_j$, $j = 1,..,N$ we use the increased demands $\xi_j - x_j + z_j$, $j = 1,\dots,N$, add the sum of the $t_j(z_j)$, $j = 1,\dots,N$ values to the cost of the flow and treat $z_1,\dots,z_N$ as nonnegative variables. Both from the theoretical and practical point of view it is desirable to chose the outage costs in such a way that $t_j(z)$ is monotonically increasing in $[0,\infty)$ further there is a fixed charge at $z = 0$, i.e.

$$t_j(z) = \begin{cases} 0 & \text{if } z = 0 , \\ \geqq T_j > 0 & \text{if } z > 0 . \end{cases}$$

Clearly we can choose $T_1,\dots,T_N$ so large that if $A(x,y,\xi)$ occurs, we obtain automatically $z_1 = \dots = z_N = 0$ in the optimal solution of the just described extension of the flow problem. Let $\mu = \mu(x,y,\xi)$ denote its optimum value.

Now we formulate our planning model in the following way

$$\text{minimize} \left[\sum_j c_j(x_j) + \sum_{h,k} c_{hk}(y_{hk}) + E(\mu)\right]$$

subject to

$$\begin{aligned} & P(A(x,y,\xi)) \geqq P , \\ (2.1) \qquad & X_j^{(\ell)} \leqq x_j \leqq X_j^{(u)} , \quad \text{all } j \\ & Y_{hk}^{(\ell)} \leqq y_{hk} \leqq Y_{hk}^{(u)} , \quad \text{all } h,k . \end{aligned}$$

3. Application of the Gale-Hoffman Theorem for the Determination of $A(x,y,\xi)$ in Terms of Linear Inequalities.

The feasibility theorems formulated by Gale resp. Hoffman concerning network flows resp. circulations are essentially the same. The presentation given by Gale suits very well to our problem. Thus we follow his way of thinking.

A network $[N,y]$ consists of a finite set of nodes N and a capacity function y defined on the product set $N \times N$, where $y(a,b)$ can take on nonnegative values and $+\infty$.

A flow i on $[N,y]$ is defined as a function on $N \times N$ satisfying the relations

$$\left.\begin{array}{l} i(a,b) + i(b,a) = 0 \; , \\ i(a,b) \leqq y(a,b) \; , \end{array}\right\} \quad \text{for all} \quad a,b \in N \; .$$

A network demand q is a real valued function on N. A negative q will be interpreted as a supply. Note that we already used the term demand concerning power systems and denoted by ξ_j the demand appearing in the area that has to be satisfied primarily by system j and by other systems only in the case if system j has no power enough to do that. In what follows ξ_j will be termed area demand (of area j) or simply demand. In our planning model the network demand at system j will be given by $\xi_j - x_j$ and this can be positive, zero or negative as well.

If S,T are subsets of N, then we define

$$q(S) = \sum_{a \in S} q(a) \quad ,$$

$$i(S,T) = \sum_{a \in S, b \in T} i(a,b) \quad .$$

Clearly $q(S)$ is an additive set function and $i(S,T)$ is additive in S and T whenever the other set is fixed. More explicitly, we have

$$q(S_1 \cup S_2) = q(S_1) + q(S_2) \quad \text{if} \quad S_1 \cap S_2 = \phi \; ,$$

$$i(S_1 \cup S_2, T) = i(S_1,T) + i(S_2,T) \quad \text{if} \quad S_1 \cap S_2 = \phi \; ,$$

$$i(S, T_1 \cup T_2) = i(S,T_1) + i(S,T_2) \quad \text{if} \quad T_1 \cap T_2 = \phi \; .$$

These definitions imply for every S and T

$$i(S,S) = 0 \quad ,$$

$$i(S,T) \leqq y(S,T) \quad .$$

The network demand q is called feasible if there exists a flow i such that

$$i(N,a) \geqq q(a) \quad \text{for every} \quad a \in N \quad .$$

This implies that

$$i(N,S) \geqq q(S) \quad \text{for every} \quad S \subset N \quad .$$

Theorem 3.1 (Feasibility theorem). The network demand q is feasible if and only if for every $S \subset N$ we have

$$q(\bar{S}) \leqq y(S,\bar{S}) \tag{3.1}$$

where $\bar{S} = N - S$.

In the above presentation it was more convenient to write $i(a,b)$, $y(a,b)$ and $q(a)$ instead of subscripted variables. Now we return to our problem of interconnected power systems and reestablish the notations i_{ab} and y_{ab}. The notation $q(a)$ or q_a will not be used. The network demands are $\xi_1 - x_1, \dots, \xi_N - x_N$; these symbols will be used in the sequel.

The feasibility theorem provides us with a system of linear inequalities expressing the event $A(x,y,\xi)$. The inequalities (3.1) give necessary and sufficient condition that the whole pool can be supplied by power. The y function give the tie line capacities.

Let us consider some examples. The first is a network consisting of two nodes (see Fig. 1). Assume that $y_{12} = y_{21}$ and let them denote simply by y.

$$\begin{array}{ccc} \xi_1 - x_1 & & \xi_2 - x_2 \\ \underset{1}{0} & \overset{}{\underset{y}{\rule{2cm}{0.4pt}}} & \underset{2}{0} \end{array}$$

Figure 1.

Then $q(1) = \xi_1 - x_1$, $q(2) = \xi_2 - x_2$ and (3.1) gives

$$\xi_1 - x_1 + \xi_2 - x_2 \leqq 0 \qquad (S \text{ is empty}) \ ,$$

$$(3.2) \qquad \xi_1 - x_1 \leqq y \qquad (S = \{2\}) \ ,$$

$$\xi_2 - x_2 \leqq y \qquad (S = \{1\}) \ .$$

If $\bar{S}$ is empty, then (3.1) holds trivially because negative values are not allowed for y.

We can give also a system of relations in terms of the bus powers $i_1 = i$ and $i_2 = -i$. In fact these have to satisfy the inequalities

$$x_1 - i \geqq \xi_1 \ ,$$

$$(3.3) \qquad x_2 + i \geqq \xi_2 \ ,$$

$$-y \leqq i \leqq y \ .$$

The inequalities (3.2) give the solvability condition - with respect to i - of the system (3.3). In other words, (3.2) is the projection of the convex polyhedron (3.3) given in R^6 onto the R^5 space of the variables $x_1, x_2, y, \xi_1, \xi_2$.

Consider now the network of three nodes illustrated in Fig. 2.

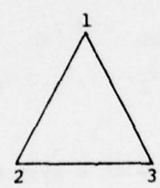


Figure 2.

Assume that the tie line capacities are symmetrical and introduce the notations $y_1 = y_{12} = y_{21}, y_2 = y_{23} = y_{32}, y_3 = y_{13} = y_{31}$. Then the relations (3.1) give the following system of linear inequalities

$$
(3.4)\qquad
\begin{aligned}
&\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 \leqq 0 \ , \\
&\xi_1 - x_1 \leqq y_1 + y_3 \ , \\
&\xi_2 - x_2 \leqq y_1 + y_2 \ , \\
&\xi_3 - x_3 \leqq y_2 + y_3 \ , \\
&\xi_1 - x_1 + \xi_2 - x_2 \leqq y_2 + y_3 \ , \\
&\xi_2 - x_2 + \xi_3 - x_3 \leqq y_1 + y_3 \ , \\
&\xi_1 - x_1 + \xi_3 - x_3 \leqq y_1 + y_2 \ .
\end{aligned}
$$

The system of linear inequalities which has to be satisfied by the bus powers i_1, i_2, $i_3 = -(i_1 + i_2)$ is given by

$$
(3.5)\qquad
\begin{aligned}
x_1 - i_1 &\geqq \xi_1 \ , \\
x_2 - i_2 &\geqq \xi_2 \ , \\
x_3 + i_1 + i_2 &\geqq \xi_3 \ , \\
-(y_1 + y_3) \leqq i_1 &\leqq y_1 + y_3 \ , \\
-(y_1 + y_2) \leqq i_2 &\leqq y_1 + y_2 \ , \\
-(y_2 + y_3) \leqq i_1 + i_2 &\leqq y_2 + y_3 \ .
\end{aligned}
$$

The convex polyhedron (3.4) is again a projection of the convex polyhedron (3.5).

The third example will be the network illustrated in Fig. 3. The tie line capacities

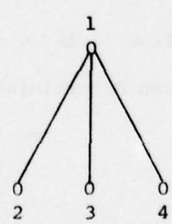


Figure 3.

are assumed to be symmetrical and will be denoted by $y_2 = y_{12}, y_3 = y_{13}, y_4 = y_{14}$. The relations (3.1) are specialized as follows:

$$
\begin{aligned}
&\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 \leqq 0 \ , \\
&\xi_1 - x_1 \leqq y_2 + y_3 + y_4 \ , \\
&\xi_2 - x_2 \leqq y_2 \ , \\
&\xi_3 - x_3 \leqq y_3 \ , \\
&\xi_4 - x_4 \leqq y_4 \ , \\
&\xi_1 - x_1 + \xi_2 - x_2 \leqq y_3 + y_4 \ , \\
&\xi_1 - x_1 + \xi_3 - x_3 \leqq y_2 + y_4 \ , \\
&\xi_1 - x_1 + \xi_4 - x_4 \leqq y_2 + y_3 \ , \\
&\xi_2 - x_2 + \xi_3 - x_3 \leqq y_2 + y_3 \ , \\
&\xi_2 - x_2 + \xi_4 - x_4 \leqq y_2 + y_4 \ , \\
&\xi_3 - x_3 + \xi_4 - x_4 \leqq y_3 + y_4 \ , \\
&\xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 \leqq y_2 + y_3 + y_4 \ , \\
&\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 \leqq y_4 \ , \\
&\xi_1 - x_1 + \xi_2 - x_2 + \xi_4 - x_4 \leqq y_3 \ , \\
&\xi_1 - x_1 + \xi_3 - x_3 + \xi_4 - x_4 \leqq y_2 \ .
\end{aligned}
\tag{3.6}
$$

The number of relations contained in (3.1) equals 2^N-1 if we disregard the trivial relation obtained in the case $S = N$. From (3.1) we can immediately conclude

<u>Theorem 3.2</u>. The linear inequalities determining $A(x,y,\xi)$ consist of inequalities of the form:

Partial sum of the ξ_j's $\leqq$

Partial sum of the x_j's + Partial sum of the y_j's .

Theorem 1 also tells us what partial sums have to be combined. We can write them automatically if all pairs are assumed to be connected with eventually zero capacity tie lines. The form of the inequalities emphasized in Theorem 2 will be important when proving the convex programming property of our optimization problem in case of a multivariate gamma distribution.

4. The Use of Multivariate Probability Distributions

A probability distribution defined on the measurable subsets of R^N is said to be logarithmically concave (logconcave) if for every pair of convex sets $A, B \subset R^N$ and $0 < \lambda < 1$ we have the inequality

$$P(\lambda A + (1-\lambda)B) \geqq [P(A)]^{\lambda}[P(B)]^{1-\lambda} . \tag{4.1}$$

If the random vector $\xi \in R^N$ has logconcave distribution, then the same holds for all linear transforms of ξ [8]. Most important in this respect are the following theorems.

Theorem 4.1. If ξ has a continuous probability distribution and its probability density function is a logconcave point function (i.e. if f denotes the density, then for every $u, v \in R^N$ and $0 < \lambda < 1$ we have $f(\lambda u + (1-\lambda)v) \geqq [f(u)]^{\lambda}[f(v)^{1-\lambda})$, then the probability distribution of ξ is logconcave i.e. (4.1) holds where

$$P(C) = P(\xi \in C)$$

for every measurable $C \subset R^N$.

Theorem 4.2. If $g_1(z,w),\dots,g_r(z,w)$ are convex functions of all variables in the entire space and ξ is a random vector having (the same number of components as w and) a logconcave probability distribution, then the function

$$P(g_1(z,\xi) \leqq 0,\dots,g_r(z,\xi) \leqq 0) \tag{4.2}$$

is a logconcave point function of the variable z.

Theorem 4.1 was proved in [7]. Theorem 4.2 easily follows from Theorem 4.1. For a proof see [7]. Further theorems of similar type are proved in [2].

The most immediate examples for logconcave distributions are the following:

a. Uniform distribution in a convex set $D \subset R^N$; the probability density equals

$$f(z) = \begin{cases} 0 & \text{if } z \notin D , \\ \frac{1}{|D|} & \text{if } z \in D , \end{cases} \tag{4.3}$$

where $|D|$ is the Lebesgue measure of D assumed to be finite;

b. Normal distribution; the probability density equals

$$f(z) = \frac{\sqrt{\operatorname{Det} C}}{(2\pi)^{N/2}} e^{-\frac{1}{2}(z-m)' C^{-1}(z-m)}, \quad z \in R^N \tag{4.4}$$

where C is the covariance matrix assumed to be nonsingular and m is the expectation vector.

We shall use a third multivariate distribution that is not necessarily logconcave. The joint probability distribution function will be logconcave, however. This is the multi-gamma distribution introduced in [10].

Let $\xi_1,\dots,\xi_N$ be gamma distributed random variables with parameters $\lambda_1, \vartheta_1; \dots; \lambda_N, \vartheta_N$ where $\lambda_j > 0$, $i = 1,\dots,N$ and $\vartheta_j \geq 0$, $j = 1,\dots,N$. Thus if $\vartheta_j > 0$, then ξ_j has the density

$$\frac{\lambda_j^{\vartheta_j - 1} z^{\vartheta_j} e^{-\lambda_j z}}{\Gamma(\vartheta_j)}, \quad z > 0 \quad . \tag{4.5}$$

If $\vartheta_j = 0$, then by definition $P(\xi_j = 0) = 1$. A gamma distribution is said to be standard if the λ parameter equals 1. The random variables $\zeta_1 = \lambda_1\xi_1,\dots,\zeta_N = \lambda_N\xi_N$ are clearly standardized. Assume that every ζ_j is a partial sum of a collection of independent, standard gamma distributed variables $\eta_1,\dots,\eta_r$, where $r = 2^N-1$, thus

$$\zeta = A\eta \quad , \tag{4.6}$$

where A is an $N \times 2^N-1$ matrix with 0, 1 entries, all columns of which are different from each other and from the zero vector. A representation technique producing the (approximate) (4.6) form for ξ is given in [10].

If a standard gamma distributed random variable η has ϑ parameter ≥ 1, then by Theorem 4.1, η has a logconcave distribution. If $\vartheta < 1$, then write

$$\eta = \left(\eta^{\vartheta}\right)^{1/\vartheta} \tag{4.7}$$

and observe that

a.) η is a convex function of η^{ϑ} ;

b.) η^{ϑ} has a logconcave density .

The first assertion is trivial. The second assertion can be proved as follows. Let $z > 0$; then the probability density function of η^{ϑ} at the point z equals

$$\text{(4.8)} \qquad \frac{d}{dz} P(\eta^{\vartheta} < z) = \frac{d}{dz} P(\eta < z^{\frac{1}{\vartheta}}) = \frac{d}{dz} \int_0^{z^{1/\vartheta}} \frac{1}{\Gamma(\vartheta)} t^{\vartheta-1} e^{-t} dt = \frac{1}{\Gamma(\vartheta+1)} e^{-z^{1/\vartheta}} .$$

The function: equal to zero if $z \leqq 0$ and equal to the right hand side expression of (4.8) if $z > 0$, is logconcave. Thus η^{ϑ} has a logconcave distribution.

Now if in (4.6) η_j has parameter $\vartheta_j < 1$, then we represent η_j in the form (4.7), otherwise we leave η_j unchanged. Then express ξ_j as $\frac{1}{\lambda} \zeta_j$, $j = 1,\ldots,N$ and substitute the thus obtained expressions for ξ_j into the inequalities determining $A(x,y,\xi)$. Using Theorem 3.2 and Theorem 4.2, we immediately obtain

<u>Theorem 4.3.</u> If $\xi_j = \frac{1}{\lambda_j} \zeta_j$, $j = 1,\ldots,N$ and ζ has the form (4.6) (where η has independent standard gamma components), then

$$P(A(x,y,\xi))$$

is a logconcave function of the variables contained in the vectors x and y.

As an example consider the case of $N = 2$. Assume that $\lambda_1 = 1$, $\lambda_2 = 1$,

$$\text{(4.9)} \qquad \begin{aligned} \xi_1 &= \eta_1 + \eta_2 \ , \\ \xi_2 &= \eta_1 \qquad + \eta_3 \ , \end{aligned}$$

where $\vartheta_1 = 2$, $\vartheta_2 = 1/2$, $\vartheta_3 = 3/2$. Substituting ξ_1 and ξ_2 into (3.2) and writing

$$\eta_2 = (\sqrt{\eta_2})^2 \ ,$$

we obtain the inequalities

$$2\eta_1 + (\sqrt{\eta_2})^2 + \eta_3 \leqq x_1 + x_2 ,$$

$$\eta_1 + (\sqrt{\eta_2})^2 \leqq x_1 + y ,$$

$$\eta_1 + \eta_3 \leqq x_2 + y .$$

The probability that all these inequalities are satisfied, is a logconcave function of x_1, x_2, y.

5. Detailed Formulation of the Problem for $N = 2$ and $N = 3$.

In case of $N = 2$ we start from Relations (3.3). We use the nonnegative variables i^+_{12} and i^-_{12} but omit the subscripts for the sake of simplicity. We have the equality

$$i = i^+ - i^- .$$

The case that at the same time $i^+ > 0$, $i^- > 0$ will be automatically discarded by the optimization principle provided $d_{12} > 0$ and $d_{21} > 0$. The second stage problem is the following:

$$\text{minimize } [d_{12}i^+ + d_{21}i^- + t_1(z_1) + t_2(z_2)]$$

subject to

$$\begin{aligned} & x_1 - i^+ + i^- + z_1 \geqq \xi_1 , \\ & x_2 + i^+ - i^- + z_2 \geqq \xi_2 , \\ & -y \leqq i^+ - i^- \leqq y , \\ & i^+, i^-, z_1, z_2 \geqq 0 . \end{aligned} \tag{5.1}$$

The first stage problem is the following special case of Problem (3.1):

$$\text{minimize } [c_1(x_1) + c_2(x_2) + c(y) + E(\mu)]$$

subject to

$$P\begin{pmatrix} \xi_1 + \xi_2 \leqq x_1 + x_2 \\ \xi_1 \leqq x_1 + y \\ \xi_2 \leqq x_2 + y \end{pmatrix} \geqq p , \tag{5.2}$$

$$X_j^{(\ell)} \leqq x_j \leqq X_j^{(u)} ,$$

$$Y^{(\ell)} \leqq y \leqq Y^{(u)},\ j = 1, 2 ,$$

where $\mu = \mu(x_1, x_2, y, \xi_1, \xi_2)$ is the (random) optimal solution of the second stage problem and p is a prescribed (very high) probability. The 12 subscripts are omitted in c, y and $Y^{(\ell)}$, $Y^{(u)}$.

In this special case the value of the objective function of the second stage problem can easily be expressed in terms of x_1, x_2, y, ξ_1, ξ_2. Four cases have to be investigated.

a.) $\xi_1 - x_1 \leqq 0$, $\xi_2 - x_2 \leqq 0$. Then $\mu = 0$.

b.) $\xi_1 - x_1 \leqq 0$, $\xi_2 - x_2 > 0$. The optimal solution is

$i^+ = \min(x_1 - \xi_1, \xi_2 - x_2, y)$, $i^- = 0$, $z_1 = 0$, $z_2 = \xi_2 - x_2 - i^+$ and $\mu = d_{12}i^+ + t_2(z_2)$.

c.) $\xi_1 - x_1 > 0$, $\xi_2 - x_2 \leqq 0$. The optimal solution is

$i^+ = 0$, $i^- = \min(\xi_1 - x_1, x_2 - \xi_2, y)$, $z_2 = 0$, $z_1 = \xi_1 - x_1 - i^-$ and $\mu = d_{21}i^- + t_1(z_1)$.

d.) $\xi_1 - x_1 > 0$, $\xi_2 - x_2 > 0$. The optimal solution is

$i^+ = i^- = 0$, $z_1 = \xi_1 - x_1$, $z_2 = \xi_2 - x_2$ and $\mu = t_1(z_1) + t_2(z_2)$.

If ξ_1, ξ_2 have a continuous joint distribution and their joint probability density function is $f(v_1, v_2)$, then the expectation of μ can be written in the following manner

$$E(\mu) = \int_{x_2}^{\infty} \int_{-\infty}^{x_1} [d_{12} \min(x_1 - v_1, v_2 - x_2, y) +$$

$$+ t_2(v_2 - x_2 - \min(x_1 - v_1, v_2 - x_2, y))]f(v_1, v_2)dv_1dv_2 +$$

$$+ \int_{-\infty}^{x_2} \int_{x_1}^{\infty} [d_{21} \min(v_1 - x_1, x_2 - v_2, y) +$$

$$+ t_1(v_1 - x_1 - \min(v_1 - x_1, x_2 - v_2, y))]f(v_1, v_2)dv_1dv_2 +$$

$$+ \int_{x_2}^{\infty} \int_{x_1}^{\infty} [t_1(v_1 - x_1) + t_2(v_2 - x_2)]f(v_1, v_2)dv_1dv_2 \quad .$$

If the network consists of three nodes, then the configuration given by Figure 2 is the most general. If somewhere there is no interconnection then we set the tie line capacity equal to zero. We start from Relations (3.5) and use the representation

$$i_1 = i_{12}^+ - i_{12}^- + i_{13}^+ - i_{13}^- \quad ,$$

(5.3)

$$i_2 = i_{12}^- - i_{12}^+ + i_{23}^+ - i_{23}^- \quad .$$

The cases $i_{hk}^{+} > 0$, $i_{hk}^{-} > 0$ will be automatically discarded in the problem provided the coefficients of these variables are positive in the objective function. The second stage problem is the following:

$$\text{minimize}[d_{12}i_{12}^{+} + d_{21}i_{21}^{-} + d_{13}i_{13}^{+} + d_{31}i_{13}^{-} + d_{23}i_{23}^{+} + d_{32}i_{32}^{-} + $$

$$+ t_1(z_1) + t_2(z_2) + t_3(z_3)]$$

subject to

$$x_1 - i_1 + z_1 \geq \xi_1 ,$$

$$x_2 - i_2 + z_2 \geq \xi_2 ,$$

$$x_3 + i_1 + i_2 + z_3 \geq \xi_3 ,$$

$$|i_1| \leq y_1 + y_3 ,$$

$$|i_2| \leq y_1 + y_2 ,$$

$$|i_1 + i_2| \leq y_2 + y_3 ,$$

where i_1 and i_2 are given by (5.3). The first stage problem is that special case of (2.1), where $A(x,y,\xi)$ means the system of linear inequalities (3.4).

6. Remarks Concerning the Numerical Solution.

We intend to return to the numerical solution of Problem (2.1) in another paper. In the special case when $E(\mu)$ is disregarded in the first stage problem, we obtain a probabilistic constrained stochastic programming problem similar to that investigated in [11]. There numerical example is presented using the normal and the multigamma distributions and the nonlinear programming method of supporting hyperplanes proposed by Veinott [13]. Of course other nonlinear programming methods can be used too.

When computing function values in the course of the optimization process, the LOLP is obtained in case of the assumed capacities. Thus, as a by-product of our theory, we obtain a method for finding the LOLP. In general the LOLP are obtained by simulation, using the inequalities defining $A(x,y,\xi)$. In some special cases it may be possible to express the LOLP more explicitly. If e.g. ξ has a discrete distribution of some simple type, then we may be able to add the probabilities belonging to those ξ values which satisfy the inequalities $A(x,y,\xi)$. This sum equals 1 - LOLP.

Conclusion

We have expressed the condition that interconnected power systems can assist each other to meet total demand in the pool in terms of linear inequalities containing the area demands, area generating capacities and tie line connection capacities as variables. This provides us with a tool for the Monte Carlo simulation of the system reliability = 1 - LOLP. Moreover, imposing a lower bound on the system reliability and minimizing total investment cost we are able to plan power generating and tie line connection capacities of interconnected power systems. A model is formulated for the same problem where the operating and outage costs are also included. In this case the numerical solution is still possible if the number of interconnected systems is small. The method of solution will be presented in a further paper.

Acknowledgement. The importance of the author's variant of two stage programming under uncertainty in planning of interconnected power systems was recognized by the author after a discussion on the subject with Professor K. Szendy to whom the author expresses his thanks.

REFERENCES

[1] R. Billinton, "Bibliography on the Application of Probability Methods in Power System Reliability Evaluation", IEEE Trans. PAS, 91 (1972) 649-660.

[2] C. Borell, "Convex Set Functions in d-Space", Periodica Mathematica Hungarica 6 (1975) 111-136.

[3] G. B. Dantzig and A. Madansky, "On the Solution of Two Stage Linear Programs under Uncertainty", in: Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability (University of California Press, Berkeley, California, 1961), 165-176.

[4] D. Gale, "A Theorem on Flows in Networks", Pacific Journal of Mathematics 7 (1957) 1073-1082.

[5] A. J. Hoffman, "Some Recent Applications of the Theory of Linear Inequalities to Extremal Combinatorial Analysis", in: Proceedings of Symposia in Applied Mathematics Vol. X. Combinatorial Analysis (AMS,1960), 113-127.

[6] P. Kall, Stochastic Linear Programming (Springer 1976).

[7] A. Prékopa, "Logarithmic Concave Measures with Application to Stochastic Programming", Acta Sci. Math. (Szeged) 32 (1971) 301-316.

[8] A. Prékopa, "Stochastic Programming Models for Inventory Control and Water Storage", in: Colloquia Math. Soc. JANOS BOLYAI 7 (North Holland Publ. Comp. 1973).

[9] A. Prékopa, "Contributions to the Theory of Stochastic Programming", Mathematical Programming 4 (1973) 202-221.

[10] A. Prékopa and T. Szántai, "A New Multivariate Gamma Distribution and its Fitting to Empirical Streamflow Data", Water Resources Research, to appear.

[11] A. Prékopa and T. Szántai, "Flood Control Reservoir System Design Using Stochastic Programming", Mathematical Programming, to appear.

[12] Ch. R. Scherer and L. Joe, "Electric Power System Planning with Explicit Stochastic Reserves Constraint", Management Science 23 (1977) 978-985.

[13] A. F. Veinott, "The Supporting Hyperplane Method for Unimodal Programming", Operations Research (1967) 147-152.

[14] R. Wets, "Programming Under Uncertainty: The Solution Set", SIAM Journal of Applied Math. 14 (1966) 1143-1151.

AP/jvs

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ~~1835~~ 14 MRC-TSR-1835 / 2. GOVT ACCESSION NO.	3. ~~RECIPIENT'S CATALOG NUMBER~~ 9 Technical
4. TITLE (and Subtitle) 6 PLANNING IN INTERCONNECTED POWER SYSTEMS: AN EXAMPLE OF TWO-STAGE PROGRAMMING UNDER UNCERTAINTY.	5. TYPE OF REPORT & PERIOD COVERED Summary Report, ~~no specific reporting period~~
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Andras Prekopa	8. CONTRACT OR GRANT NUMBER(s) 15 DAAG29-75-C-0024
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 5 - Mathematical Programming and Operations Research
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709	12. REPORT DATE 11 March 1978 12 27p.
	13. NUMBER OF PAGES ~~23~~
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Stochastic optimization, Logconcave measures, Power systems.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The problem of planning in interconnected power systems is formulated as a stochastic programming model, a variant of the model: two stage programming under uncertainty. In ~~our~~ this case the solvability of the "second stage problem" is required only for a probability near unity.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 221200 B

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)